Name: ________________________________ Date: ___________________________________

PSTAT 5A: Homework 5

Summer Session B 2025, with *Annie Adams*

1. A farmer claims the average weight of their avocados is 250 grams. A quality control inspector wants to test this claim. The inspector takes a random sample of 35 avocados and finds that the average weight is 242 grams with a sample standard deviation of 28 grams. The inspector wants to perform a hypothesis test at a 5% level of significance to determine if the true average avocado weight is different from the farmer's claim.

(a) Define the parameter of interest.

The true population mean weight of the farmer's avocados, μ.

(b) State the null and alternative hypotheses using proper notation.

$$H_0 : \mu = 250$$

$$H_a : \mu \neq 250$$

(c) What distribution should be used for this test? Be sure to check any relevant conditions.

The sample size n = 35 is large enough for CLT to apply, so the sample sampling distribution of the sample mean will be approximately normal. Since the population standard deviation, σ , is unknown, we must use the sample standard deviation, s, which necessitates the use of the t- distribution.

Thus, we should use the t distribution with n -1 = 35-1 ⇒ t_{34}.

(d) Compute the observed value of the test statistic.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{242 - 250}{28/\sqrt{35}} \approx -1.696$$

(e) What is the critical value of the test?

For a two tailed test with $\alpha = 0.05$ and df = 34, we find that the critical value is approximately 2.03 .

(f) Based on the results, what is your conclusion? Phrase your answer in the context of the problem.

We reject our null hypothesis if |TS| > c. Since |ts| = 1.696, which is not greater than 2.03, we fail to reject the null hypothesis. At an $\alpha = .05$, we fail to reject the null hypothesis that the average weight of the avocados is 250 grams.

2. A mountain bike trail organization claims that their championship downhill course can be completed by professional riders in an average of 75 seconds. A veteran rider suspects that due to recent trail modifications making it more challenging, riders are now taking longer than the advertised time to complete the course. However, a rival rider argues that experienced bikers are actually getting faster and completing it in less time than claimed. To settle this debate, they analyze the run times from 50 recent qualifying runs and find an average completion time of 73.6 seconds with a sample standard deviation of 4.8 seconds. Conduct a hypothesis test at the 1% level of significance to determine if there is sufficient evidence that riders are completing the course in less time than the organization's claim.

(a) Define the parameter of interest

μ = true average completion time (in seconds) for professional mountain bike rides on the champion downhill course

(b) State the null and alternative hypotheses with proper notation

$$H_0 : \mu = 75$$

$$H_A : \mu < 75$$

(c) What distribution should be used for this test? Be sure to check any relevant conditions.

The sample size n = 50 is large enough for CLT to apply, so the sample sampling distribution of the sample mean will be approximately normal. Since the population standard deviation, σ , is unknown, we must use the sample standard deviation, s, which necessitates the use of the t- distribution.

Thus, we should use the t distribution with n -1 = 50-1 ⇒ t_{49}.

(d) Compute the observed value of the test statistic.

$$ts = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{73.6 - 75}{4.8/\sqrt{50}} = -2.06$$

(e) Conduct the test using critical values

$\alpha = .01 \ df = 49 \ t_{.01,49} = -2.405$

Reject H_0 if $t < -2.405$ ⇒ Fail to Reject H_0

(f) Conduct the test using a p value

$\mathbb{P}(TS < -2.06) = .022$

Reject H_0 if p val < α ⇒ if $.022 < .01$ ⇒ Fail to reject H_0

(g) Based on the results, what is your conclusion? Make sure your conclusion includes: the significance level, the decision of the test as a function of the null hypothesis (in affirmation or not in affirmation of the alternative hypothesis) See lecture 14 if you need help!

At an α = .01 level of significance, there was not sufficient evidence to reject the claim that professional mountain bikers are completing the downhill course in 75 seconds in favor of the alternative that professional mountain bikers are completing the course in less than 75 seconds.

3. A researcher wants to compare the effectiveness of two different fertilizers on plant growth. She grows 25 plants using Fertilizer A and 28 plants using Fertilizer B. After a month, she measures the height of each plant (in centimeters). The results are summarized below:

	Sample Mean ($\bar{x}$)	Sample Std. Dev (s)
Fertilizer A	35.2 cm	4.1 cm
Fertilizer B	32.5 cm	3.8 cm

Assume the necessary independence and normality assumptions are met. The researcher wants to test if there's a significant difference in the average plant height between the two fertilizers using a 5% significance level.

(a) Define the parameters of interest.

The true difference between the population mean heights of plants grown with Fertilizer A and plants grown with fertilizer B.

(b) State the null and alternative hypotheses.

$$H_0 : \mu_1 = \mu_2$$

$$H_a : \mu_1 \neq \mu_2$$

(c)Compute the observed value of the test statistic.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(35.2 - 32.5) - 0}{\sqrt{\frac{4.1^2}{25} + \frac{3.8^2}{28}}} \approx \frac{2.7}{\sqrt{0.6724 + 0.5157}} = \frac{2.7}{\sqrt{1.1881}} \approx 2.48$$

(d) What is the approximate distribution of the test statistic under the null hypothesis? Be sure to include the relevant parameter.

The approximate distribution is a t- distribution.

$df = min(n_1 - 1, n_2 - 1) = min(24,27) = 24$

So the distribution is t_{24}.

(e) What are the critical values of the test?

For a two tailed test with $\alpha = 0.05$ and $df = 24$, the critical values are

$t_{0.025,24}$ and $-t_{0.025,24}$. Using a t distribution table, the critical values are approximately ± 2.064.

(f) What is your conclusion? Phrase your answer in the context of the problem.

The observed test statistic t $t \approx 2.48$ is outside the critical values (-2.064, 2.064). Therefore, we reject the null hypothesis. There is sufficient evidence at the 5% significance level to conclude that there is a significant difference in the average plant height between the two fertilizers.

4. A random sample of 60 small businesses in Santa Barbara had an average monthly revenue of \$135,000, with a standard deviation of \$32,000. A similar random sample of 65 small businesses in Montecito had an average monthly revenue of \$115,200 with a standard deviation of \$25,500.

Calculate the 90% confidence interval for the difference in the population mean monthly revenue for small businesses between the two cities.

These are both random samples so the observations with each sample are independent of one another. Given $n_1 = 60 \geq 30$ and $n_2 = 65 \geq 30$, we have a large enough sample to use CLT.

$$df = min(n_1 - 1, n_2 - 1) = min(59, 64) = 59$$

Calculate t critical value given $\alpha = 0.1$ which is 1.68.

$$\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$= (135000 - 115200) \pm 1.671 \sqrt{\frac{32000^2}{60} + \frac{25500^2}{65}}$$

$$=19800 \pm 8693 = [\$11{,}107, \$28{,}493]$$

We are 90% confident that the true difference in mean monthly revenue for small businesses in Santa Barbara and small businesses in Montecito is between \$11,107 and \$28,493.