Name: _____ Date: _____

PSTAT 5A: Homework 6 (Extra Credit)

Summer Session B 2025, with *Annie Adams*

1. A retail company wants to test whether a new customer service training program *increases* customer satisfaction ratings. They randomly select 40 employees who interact directly with customers. Each employee's performance is measured using customer satisfaction surveys: once before the training program and once after completing the training program.The differences in customer satisfaction ratings (satisfaction after training minus satisfaction before training) were calculated. The mean difference was found to be 8.425 points, with a standard deviation of 15.892 points.

   (a) Conduct a 90% confidence interval for the mean difference in customer satisfaction ratings.

   $\boxed{[4.205, 12.645]}$

   (b) Conduct a one-sided hypothesis test at a 10% significance level to determine if the customer service training program significantly increases customer satisfaction ratings. Be sure to phrase your conclusion clearly and in the context of the problem.

   At an $\alpha = .10$ level of significance, we reject the null hypothesis that the mean difference in customer satisfaction before and after implementing the customer service training program is the same. This data suggests that the mean difference in customer satisfaction after implementing the customer service training program is greater than that before.

2. According to the National Center for Education Statistics (NCES), 87% of college students in the United States have reliable high-speed internet access in their living situations. A student services administrator at a regional university suspects that internet access rates among students at their institution may be different from the

national average, particularly given the mix of on-campus, off-campus, and commuter students.

To investigate this claim, the administrator surveys a random sample of 240 students at their university and finds that 196 students report having reliable high-speed internet access in their living situations.

(a) Conduct a hypothesis test to determine whether the proportion of students with reliable internet access at this university differs significantly from the national average. Use a significance level of α = 0.05.

At an $\alpha = .05$ level of significance, we reject the null hypothesis that the proportion of students with reliable internet access at this university does not differ from the national average, in favor of the alternative hypothesis that the proportion of students with reliable internet access at this university does differ from the national average

(b) Construct a 95% confidence interval for the true proportion of students with reliable internet access at this university.

$\boxed{[.77, .87]}$

3. A psychology professor is investigating the relationship between study time (in hours) and performance on a statistics exam (scored out of 100 points). Letting **x** denote **study time** (the explanatory variable) and **y** denote **exam score** (the response variable), a sample of 85 students yielded the following results:

$$\sum_{i=1}^{85} x_i = 637.5 \qquad \sum_{i=1}^{85} y_i = 6,545.2 \qquad \sum_{i=1}^{85} (x_i - \bar{x})^2 = 428.75$$

$$\sum_{i=1}^{85} (x_i - \bar{x})(y_i - \bar{y}) = 892.34 \qquad \sum_{i=1}^{85} (y_i - \bar{y})^2 = 2,847.6$$

Suppose it is known that:

$\text{Var}(\hat{\beta}_1) \approx 0.00734$

(a) Find the equation of the OLS regression line

$\hat{y} = 61.39 + 2.08x$

(b) Construct a 95% confidence interval for β_1, the true change in exam score associated with a one-hour increase in study time.

$\boxed{[1.91, 2.25]}$

4. Two events A and B are such that $\mathbb{P}(A) = 3/8$, $\mathbb{P}(B) = \frac{5}{12}$, and $\mathbb{P}(B \mid A) = 1/2$.

(a) Find $\mathbb{P}(A \cup B)$

$\frac{29}{48}$

(b) Find $\mathbb{P}(A \mid B)$

$\frac{9}{20}$

5. If $Z \sim \mathcal{N}(5, 2.2)$, compute $\mathbb{P}(Z \leq 8)$.

.9131

6. A marketing research firm discovers that 73% of smartphone users check their phone within the first 10 minutes of waking up. A behavioral psychologist decides to investigate this claim by surveying a random sample of 200 smartphone users and recording the proportion who check their phone within 10 minutes of waking.

(a) Identify the population.

All smartphone users.

(b) Identify the sample

The 200 smartphone users surveyed in the sample.

(c) Define the random variable of interest.

The proportion of smartphone users in our sample of n = 200 who check their phone within 10 minutes of waking up.

(d) What is the probability that between 70% and 76% of people in the sample check their phone within 10 minutes of waking up?

66.3%

(e) What is the probability that the sample proportion differs from the true population proportion by more than 4 percentage points?

79.6%

7. As a public health researcher, Dr. Lloyd wants to estimate the true proportion of adults in her county who have received a flu vaccination this season. To investigate this, she conducts a representative survey of 180 adults and finds that 64% of those sampled have received their flu vaccination.

Construct a 92% confidence interval for the true proportion of adults in the county who have received a flu vaccination this season.

$\boxed{[.5774..7026]}$

8. *QuickFix Auto Repair* advertises that their standard oil change service takes an average of 20 minutes to complete. Skeptical of this claim, automotive journalist Sarah collects data from a random sample of 35 oil change appointments and finds that the actual average service time was 24 minutes, with a standard deviation of 8 minutes. She believes the true average service time is longer than advertised and decides to conduct an upper-tailed hypothesis test at an $\alpha = 0.05$ level of significance.

Conduct the hypothesis test and state your conclusions about QuickFix's advertising claim.

At an $\alpha = .05$ significance level, we reject the null hypothesis that the average standard oil change service takes 20 minutes, in favor of the alternative that the average standard oil change service takes more than 20 minutes.

9. *QuickFix Auto Repair* advertises that their standard oil change service takes an average of 20 minutes to complete. Skeptical of this claim, automotive journalist Sarah collects data from a random sample of 35 oil change appointments and finds that the actual average service time was 24 minutes. From industry data, it is known that oil change service times have a population standard deviation of 7 minutes. She believes the true average service time is longer than advertised and decides to conduct an upper-tailed hypothesis test at an α = 0.05 level of significance.

Conduct the hypothesis test and state your conclusions about QuickFix's advertising claim.

At an $\alpha = .05$ significance level, we reject the null hypothesis that the average standard oil change service takes 20 minutes, in favor of the alternative that the average standard oil change service takes more than 20 minutes.

10. What is the difference between question 8 and question 9? How does this difference change the distribution?

Question 8 uses a sample standard deviation, while Question 9 uses the population standard deviation. Thus, question 8 uses a t distribution and question 9 uses the normal distribution.